STATISTICS I - 2nd Year Management Science BSc - 1st semester – **05/11/2015**

1st Mid-Term Exam – Theoretical Part V1

(theoretical part duration – 20 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (40 points). This answer sheet will be collected 20 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name:____Section:_____Number:____

Each of the following 2 groups of multiple-choice questions is worth 10 points. Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade obtained in each of the 2 groups varies between a minimum of zero and a maximum of 10 points.

Indicate whether the following statements are true (**T**) or false (**F**) by ticking the corresponding box with a cross(X):

1. Let A, B, C be events of a sample space Ω.

	T	F
If A, B are mutually exclusive events and $P(B) > 0$, then $P(A\|B) = P(B)$.		
$P(A) = P(A - B) + P(A \cap B)$.		
If $B = A \cup C$, then $P(B) \geq P(C)$		
Let events A_1, A_2 and A_3 be such that $P(A_1) = 0.4, P(A_2) = 0.2, P(A_3) = 0.3$ and $P(A_i \cap A_j) = 0 \;\; i,j = 1,2,3$. Then A_1, A_2 and A_3 are a partition of sample space S.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
$F_X(x) < P(X \leq x)$ for any $x \in \mathbb{R}$		
Let $Y = \varphi(X)$ be a function of X. If X is a continuous random variable, then Y is a continuous random variable.		
If $F_X(x)$ is differentiable at x, then we have that $F'_X(x) \geq 0$		
If X is discrete, for any $a, b \in \mathbb{R}, a < b$, $P(a \leq X \leq b) = F_X(b) - F_X(a)$.		

The following question is worth 15 points and should be answered in the space provided. Justify all your steps.

6. If A and A' are complementary events, using the postulates of the measure of probability, show that $P(A') = 1 - P(A)$.

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Each of the following 2 groups of multiple-choice questions is worth 10 points. Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade obtained in each of the 2 groups varies between a minimum of zero and a maximum of 10 points.

Indicate whether the following statements are true (**T**) or false (**F**) by ticking the corresponding box with a cross(X):

1. Let A, B, C be events of a sample space Ω.

	T	F
If A, B are independent events and $P(B) > 0$, then $P(A\|B) = P(B)$.		
$P(A - B) = P(A) - P(A \cap B)$.		
If $A = B \cup C$, then $P(A) \geq P(B)$		
Let events A_1, A_2 and A_3 be such that $P(A_1) = 0.5, P(A_2) = 0.2, P(A_3) = 0.3$ and $P(A_i \cap A_j) = 0 \;\; i, j = 1, 2, 3$. Then A_1, A_2 and A_3 are a partition of sample space S.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
$F_X(x) > P(X \leq x)$ for any $x \in \mathbb{R}$		
Let $Y = \varphi(X)$ be a function of X. If X is a discrete random variable, then Y can be a continuous random variable.		
If $F_X(x)$ is differentiable at x, then we have that $F_X'(x) \geq 0$		
If X is discrete, for any $a, b \in \mathbb{R}, a < b$, $P(a < X < b) = F_X(b) - F_X(a)$.		

The following questions is worth 15 points and should be answered in the space provided. Justify all your steps.

6. If A and A' are complementary events, using the postulates of the measure of probability, show that $P(A') = 1 - P(A)$.

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STATISTICS I - 2nd Year Management Science BSc - 1st semester – **05/11/2015**

1st Mid-Term Exam – Practical Part

(practical part duration – 40 minutes)

This is Part 2: 12 marks. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the space provided.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____
____Nº:____________

Don't write here

1a.(10)	2a.(10)	3a.(11)	T:
1b.(10)	2b.(15)	3b.(15)	P:
_______	_______	_______	_______

1

Consider a city where only two daily newspapers are printed, newspaper A and newspaper B. It is known that 5% of its inhabitants read both newspapers, while 25% only read newspaper A, and 20% only read newspaper B.

a) If 20 persons were randomly chosen from the people in this city, compute the probability that exactly 4 of them read both newspapers. (signal with an X the right answer,)

(i) 0,0746 ☐ (ii) 0,9885☐ (iii) 0,0133 ☐ (iv) 0,9974 ☐

b) One person is randomly chosen from the people in this city and he\she is a reader of newspaper A. Determine the probability that the chosen person was a reader of newspaper B.

Answer to 1.b)

Let (X,Y) be a random vector representing, for a family living in a certain district, the number of children (X) and the number of rooms in their home (Y). The joint probability function is given in the following table:

Y \ X	0	1	2	3
2	0,04	0,05	0.02	0.00
3	0,05	0,09	0,14	0.05
4	0,02	0.12	0.22	0.20

a) If a family from this district have more than 1 child, find the probability that the family lives in a home with at least 3 rooms.

(i) 0,21 ☐ (ii) 0.35 ☐ (iii) 0.81 ☐ (iv) 0.46 ☐

b) Find the probability that a family from this district lives in a home with more than 1 but less than four rooms.

Answer 2.b)

3

Consider a random vector $(X,Y)$with probability density function given by:

$$f_{X,Y}(x,y) = 2 \quad (0 < x < 1; 0 < y < 1/2)$$

a) Find the marginal probability density function of X and Y. Are X and Y independent?

Answer 3.a)

b) Compute $P(X \leq 1/2)$.

Answer 3.b)

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Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____

____Nº:_____________

Don't write here

1a.(10)	2a.(10)	3a.(11)	T:
1b.(10)	2b.(15)	3b.(15)	P:
_______	________	________	________

1

Consider a city where only two daily newspapers are printed, newspaper A and newspaper B. It is known that 5% of its inhabitants read both newspapers, while 25% only read newspaper A, and 20% only read newspaper B.

c) If 10 persons were randomly chosen from the people in this city, compute the probability that exactly 2 of them read both newspapers. (signal with an X the right answer,)

(i) 0,0746 ☐ (ii) 0,9885☐ (iii) 0,0133 ☐ (iv) 0,9974 ☐

d) One person is randomly chosen from the people in this city and he\she is a reader of newspaper B. Determine the probability that the chosen person was a reader of newspaper A.

Answer to 1.b)

Let *(X,Y)* be a random vector representing, for a family living in a certain district, the number of children (X) and the number of rooms in their home (Y). The joint probability function is given in the following table:

Y \ X	0	1	2	3
2	0,04	0,05	0.02	0.00
3	0,05	0,09	0,14	0.05
4	0,02	0.12	0.22	0.20

c) If a family from this district lives in a home with 3 rooms, find the probability that the family have less than 2 children.

(i) 0,42 ☐ (ii) 0.14☐ (iii) 0.27 ☐ (iv) 0.15 ☐

d) Find the probability that a family from this district has a number of children equal or bigger than 1 but less than 3.

Answer 2.b)

3

Consider a random vector $(X, Y)$with probability density function given by:

$$f_{X,Y}(x, y) = 2 \quad (0 < x < 1; 0 < y < 1/2)$$

c) Find the marginal probability density function of X and Y. Are X and Y independent?

Answer 3.a)

d) Compute $P(Y \leq 1/4)$.

Answer 3.b)